551.509 : 551.574

SECTION II.—GENERAL METEOROLOGY.

PREDICTING MINIMUM TEMPERATURES.[1]

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With the development of orchard-heating methods and the protection of general fruit and garden crops from damage by frosts or low temperatures by heating, covering, or flooding, it becomes increasingly important that close predictions be made of the lowest temperatures that will probably be experienced in the immediate vicinity of the fruit buds or blossoms and garden truck.

The general predictions of light, heavy, or killing frosts for considerable areas are now accurately made from the morning weather map, and this information is being widely distributed by the Weather Bureau and made good use of by the people interested. During the critical period of fruit buds and blossoms and of early truck and garden crops, however, the men who are prepared to protect by heating or flooding wish a more definite statement of the probable minimum temperature, even though the information can reach them only a few hours in advance of its occurrence. This is particularly true in protecting cranberries by flooding because in order to properly flood the bogs the gates must be opened from two to six hours before the expected temperatures occur.

In an attempt to meet this demand for minimum temperature forecasts the writer, while in Ohio, made special use of two factors in making predictions in the evening for the minimum temperature on the following morning, and has since developed a third method. These are all for use on comparatively clear and still nights where there are wide local differences in temperature due to variations in topography. On cloudy or stormy nights or when strong winds prevail, accurate minimum temperature forecasts can be made only by a study of the daily weather maps.

PREDICTING MINIMUM TEMPERATURES ON CLEAR, STILL NIGHTS.

(1) *Predicting minimum temperatures by the usual or average fall in temperature on clear, still afternoons and evenings after the maximum temperature of the day is known.*—The temperature range from the maximum of one day to the minimum of the next morning is greatest in the valleys and least on the hillsides at the level of the so-called thermal belt or the stratum of warmest air of the temperature inversion that forms in valleys by the close of every clear, still night. It varies with different months, but is remarkably uniform under similar topographic conditions and at similar seasons of the year.

(2) *Predicting the minimum temperature from the median-temperature hour.*—This method was fully explained in the MONTHLY WEATHER REVIEW, October, 1914, **42**:581 and 582. In brief, it is based on the fact that the halfway temperature from the maximum of one day to the minimum of the following morning in comparatively still and clear weather occurs at about the same hour each evening. At Delaware, Ohio, during May, 1913, the average hour at which the halfway temperature occurred was 7:36 p. m.; and in May, 1914, it was 7:35 p. m. The earliest hour on any day in either of the two months was 7:15 p. m., and the latest was 7:50 p. m. This shows that at no time was the median-temperature hour more than 20 minutes from the average for the two successive years.

In determining the median-temperature hour it is necessary to keep a thermograph in operation for at least two years and to have the time factor carefully marked and the extreme records checked by means of accurate maximum and minimum thermometers. After the median-temperature hour has been determined, it is only necessary to read the temperature at this hour, subtract it from the maximum temperature of the day and then subtract this difference from the temperature prevailing at this hour. For example: if the highest temperature during the day is 68°F. and at 7:36 p. m., it has fallen to 50°, the difference, or 18 degrees, subtracted from 50° leaves 32° as the minimum temperature during the coming night. The average time of this median temperature, even under conditions of clear skies and still air, will vary slightly at different seasons of the year and in different localities.

In central Ohio, outside of the cities, the average time of the evening median will be at about 7:15 in April; 7:30 p. m. in May and June; 6:30 p. m. in September; and close to 6 p. m. in October and November. In July it is about 7:30 p. m. and in August 7 p. m. So far as we are able to determine it is not far from 6:30 p. m. in December; and 7 p. m. in January, February, and March, although our observations and studies are not very complete in the winter months.

If a strong wind is blowing in the afternoon or if it remains cloudy or partly cloudy until evening and then clears off, the time of the median will be from 30 to 45 minutes later than the average above given. If it should cloud up during the night after a clear afternoon and evening, the minimum will not be as low as indicated by the median. Records that are at hand indicate that the average time of the median will be slightly later in the valleys than at higher elevations. At Columbus, which represents a city station, the average time of the median is later than at Delaware, Ohio, and the variations between the earliest and latest hours is greater; but the error made by predicting the minimum from the average median is only slightly greater than at Delaware.

(3) *Predicting the minimum temperature from the dewpoint and relative humidity in the late afternoon.*—Neglecting certain important considerations, it is often asserted that the minimum temperature during clear, still, nights should not be much lower than the evening dewpoint temperature, but in actual practice there are found to be wide variations between these values. In the fruit-frost work in Ohio in 1914 it was found that at part of the fruit stations the minimum temperature on clear, still nights was always lower than the dewpoint of the evening before, and in some cases it was more than 20 degrees lower; at other stations the minimum varied from 10 degrees lower to 10 or more degrees higher than the evening dewpoint.

In the spring of 1915 an earnest effort was made to ascertain, if possible, under just what conditions of the atmosphere these variations occurred so that they might be anticipated. To accomplish this, observations were made at 12 different points in Ohio both morning and evening. The observations were made between 5 and 7 a. m. and 5 and 7 p. m. and covered the maximum and minimum temperatures, dry- and wet-bulb temperatures, wind direction and approximate velocity, rainfall, and

[1] A brief historical note by Prof. C. F. Marvin, appears at the end of this communication.

state of weather. Standard instruments were used, properly exposed, and in nearly every case the location was in the open country, where radiation conditions are good.

There were 28 nights in the spring of 1915 from March to June, inclusive, when the conditions were favorable for free radiation, and while all of the stations were not in operation throughout the entire period there were, at the 12 stations, a total of 222 cases when evening and morning observations were made when the night was clear and comparatively still. The records were carefully tabulated and checked, and I believe that the number of observations, the accuracy of the instruments, and the good exposure make the conclusions of marked value.

A careful study was made of the lowering of the dewpoint, the relation between the state of the weather and the depression of the minimum temperature below the evening dewpoint, and the relation between the relative humidity in the evening and morning without practical results.

THE AMOUNT OF MOISTURE IN THE ATMOSPHERE A MEASURE OF THE VARIATION OF THE MINIMUM TEMPERATURE FROM THE EVENING DEWPOINT.

It was found, however, that when there was a relatively high percentage of moisture in the atmosphere at the evening observation, the depression of the minimum temperature below the dewpoint was greater than when the moisture content was low. While this was shown in both the dewpoint and the relative humidity, the latter was used in discussing the matter. The evening relative humidity at each station on the nights under discussion was averaged, as well as the depression of the minimum below the dewpoint, and the results of the comparison for all the observations are shown in Table 1.

TABLE 1.—*Mean depression of the minimum temperature below the evening dewpoint, with stated variations of the relative humidity from the average.*

Relative humidity variations.	Number of cases.	Mean variation of minimum from dewpoint.
		°F.
More than 25 per cent above the average	6	−11.0
From 21 per cent to 25 per cent above the average	7	−8.0
From 16 per cent to 20 per cent above the average	13	−7.8
From 11 per cent to 15 per cent above the average	21	−6.5
From 6 per cent to 10 per cent above the average	25	−8.5
From 1 per cent to 5 per cent above the average	34	−5.8
From 0 per cent to 5 per cent below the average	41	−2.2
From 6 per cent to 10 per cent below the average	21	−1.9
From 11 per cent to 15 per cent below the average	25	−2.3
From 16 per cent to 20 per cent below the average	15	−0.2
From 21 per cent to 25 per cent below the average	7	+0.1
More than 25 per cent below the average	5	+4.8

This was further summarized and shown by Table 2.

TABLE 2.—*Table for estimating the variation between the evening dewpoint and the following minimum temperature, on comparatively clear still nights.*

When evening relative humidity is—	The minimum temperature will be—
More than 26 per cent below the average	5° higher than the dewpoint.
16 per cent to 25 per cent below the average	Just the same as the dewpoint.
0 per cent to 15 per cent below the average	2° lower than the dewpoint.
1 per cent to 15 per cent above the average	7° lower than the dewpoint.
16 per cent to 25 per cent above the average	8° lower than the dewpoint.
More than 25 per cent above the average	11° lower than the dewpoint.

In these calculations the average relative humidity is the average at each station in the spring of 1915 when radiation conditions prevailed. It varies for different stations from 53 to 74 per cent, and the variation from the average was obtained for each station separately. The average of all the stations is 64 per cent, therefore as a working basis Table 3 gives the approximate measure of the relation between the factors.

TABLE 3.—*Average relation between the evening relative humidity and the variation of the minimum temperature from the evening dewpoint.*

When evening relative humidity is—	The minimum will be—
Below 40 per cent	5° above the evening dewpoint.
From 40 per cent to 50 per cent	Close to the evening dewpoint.
From 50 per cent to 65 per cent	2° below the evening dewpoint.
From 65 per cent to 80 per cent	6° to 8° below the evening dewpoint.
Over 80 per cent	8° to 12° below the evening dewpoint.

It must be remembered that these values are averages and that there will be more or less variation from them, but they will enable one to make much closer minimum temperature forecasts than can be done by a consideration of the dewpoint alone.

Table 4 illustrates in detail the relation of these two factors at a few of the stations.

TABLE 4.—*Relation of the variation of the minimum temperature from the previous evening dewpoint to the evening relative humidity at five stations in 1915.*

Delaware, Ohio.		Germantown, Ohio.		Marietta, Ohio.		Warren, Ohio.		Wooster, Ohio.	
1	2	1	2	1	2	1	2	1	2
−12	10	−13	+20	−17	10	−7	0	−13	11
−12	1	−12	14	−15	19	−7	14	−13	32
−11	6	−8	12	−15	10	−7	15	−10	−3
−11	22	−7	14	−15	10	−6	16	−10	10
−11	2	−6	11	−14	24	−5	19	−9	13
−9	5	−5	0	−14	10	−5	1	−8	18
−9	4	−5	0	−10	16	−3	14	−7	23
−9	4	−4	−1	−9	0	−4	15	−7	5
−9	2	−4	0	−9	−14	−3	10	−7	0
−8	5	−4	1	−8	10	−3	12	−7	13
−8	5	−3	10	−8	−9	−2	25	−6	9
−8	−5	−3	17	−7	−1	−2	−4	−5	5
−8	−4	−3	11	−6	−17	0	−16	−5	−1
−7	−3	−2	−6	−6	−13	0	−15	−5	5
−6	−11	−1	0	−6	−5	0	−12	−4	−13
−6	−3	−1	−4	−6	−11	+1	13	−4	−9
−5	−11	−1	−6	−4	4	1	−4	−3	1
−5	6	0	−6	−2	3	2	−19	−3	−14
−4	3	+1	−14	−2	−21	2	5	−1	1
−4	7	2	−9	+10	−23	2	11	−1	−3
−3	−1	2	−2	......	......	3	−14	−1	4
−3	3	3	−16	......	......	3	−2	−1	−14
−3	8	4	−18	......	......	3	−23	0	−8
−2	3	7	−23	......	......	13	−28	0	−14
−2	−17	10	−15	......	......	......	......	+1	−11
−2	−5	......	......	......	......	......	......	4	−8
−1	−8	......	......	......	......	......	......	5	−15
+1	−26	......	......	......	......	......	......	10	−19

Column 1 is the variation of the minimum temperature from the evening dewpoint. These figures are arranged so that the greatest depression below the dewpoint is shown at the top of each column and the date with the minimum temperature highest above the dewpoint at the bottom of the column.

Column 2 gives the variation of the evening relative humidity from the average. The dates are the same as in column 1, but the dates are not given because each station is independent of the others as regards dates.

An inspection of Table 4 shows at once that the relative humidity figures that are most above the average at that station are in the upper part of column 2, in connection with the figures in column 1 showing the greatest depression of the minimum temperature below the evening dewpoint, and that the low relative humidity values are mostly at the bottom of the columns, where the minimum temperature is nearest to or even slightly above the evening dewpoint.

CORRELATION.

To show that there is a close relation between the two factors given in Table 4, the relative humidity as observed in the evening has been compared with the departure of the minimum temperature observed the following morning from the evening dewpoint at Germantown, Ohio, for those nights in the spring of 1915 when conditions favored radiation. This comparison is given in Table 5.

The method of correlation here employed is one of the more simple and has been used quite extensively by the author in the comparison of weather factors and crop yields.[3]

Prof. C. F. Marvin has published a discussion of the method in the MONTHLY WEATHER REVIEW, October, 1916, 44:551–569. Therefore, it seems unnecessary to give here an extended discussion of the method.

In explanation of Table 5 it may be stated that column R gives the relative-humidity values, and that headed Y gives the difference between the morning minimum temperature and the dewpoint temperature of the preceding evening. The column x is the departure of the humidity values from the base number 53, and y is the departure of the values of Y from the base number -2; x^2 and y^2 explain themselves, as does also xy.

TABLE 5.—*Correlation of the relative humidity as observed in the evening, and the depression of the minimum temperature, recorded the following morning, from the evening dewpoint. Data for Germantown, Ohio, for those dates in the spring of 1915, when conditions favored radiation.*

Date.	Evening relative humidity.			Departure of morning minimum from preceding evening's dewpoint.			xy
	R	x	x^2	Y	y	y^2	
1915.	%			°F.			
Mar. 12	47	−6	36	0	2	4	−12
13	35	−18	324	4	6	36	−108
27	53	0	0	−1	1	1	0
Apr. 3	53	0	0	−5	−3	9	0
7	44	−9	81	2	4	16	−36
13	52	−1	1	−4	−2	4	2
18	30	−23	529	7	9	81	−207
24	51	−2	4	2	4	16	−8
25	63	10	100	−5	−3	9	−30
May 1	53	0	0	−4	−2	4	0
9	65	12	144	−8	−6	36	−72
10	47	−6	36	−2	0	0	0
22	67	14	196	−12	−10	100	−140
24	67	14	196	−7	−5	25	−70
June 4	73	20	400	−13	−11	121	−220
5	70	17	289	−3	−1	1	−17
9	53	0	0	−5	−3	9	0
15	64	11	121	−6	−4	16	−44
16	64	11	121	−3	−1	1	−11
17	54	1	1	−4	−2	4	−2
23	39	−14	196	1	3	9	−42
24	38	−15	225	10	12	144	−180
25	49	−4	16	−1	1	1	−4
26	47	−6	36	−1	1	1	−6
27	37	−16	256	3	5	25	−80
Sums (n=25)	1,315	−10	3,308	−55	−5	673	−1,287
Base Nos	53			−2			
Mean	52.6			−2.2			

[3] MONTHLY WEATHER REVIEW, May, 1911, 39: 792–795; February, 1914, 42: 78–93; May, 1915, 43: 222–226.

Substituting the proper values in the following formulæ we get these results:

Standard deviation of relative humidity,

$$\sigma_x = \sqrt{\frac{[x^2] - \frac{[x]^2}{n}}{n}} = 11.50$$

Standard deviation of departure of minimum temperature from evening dewpoint,

$$\sigma_y = \sqrt{\frac{[y^2] - \frac{[y]^2}{n}}{n}} = 5.185$$

Coefficient of correlation,

$$r = \frac{[xy] - \frac{[x]\,[y]}{n}}{n\,\sigma_x\,\sigma_y} = -0.865$$

Probable error of correlation coefficient,

$$E_r = \pm 0.6745\,\frac{1-r^2}{\sqrt{n}} = \pm 0.034$$

Therefore,

$$r = -0.865 \pm 0.034.$$

This high value for r indicates a close relation between the factors considered. The negative sign of the coefficient shows that when the relative humidity is high, the minimum temperature will fall the greatest amount below the dewpoint of the evening, and that when the relative humidity is low, the minimum temperature will be above the dewpoint by the greatest amount.

Similar data for 9 stations in Ohio, 6 stations in the Walla Walla district of Washington and Oregon, and for the cranberry station at Mather, Wis., were discussed in the same manner and corresponding correlation coefficients computed. The results are summarized in Table 6 hereunder.

TABLE 6.—*Correlation coefficients with corresponding probable error values for the stations in the Ohio district, and others; also coefficients for the Ohio district as a whole, and for the Walla Walla, Wash., district.*

Stations.	n	r	E_r
Clyde, Ohio	23	−0.429	±0.170
Germantown, Ohio	25	−0.865	±0.034
Green Hill, Ohio	28	−0.388	±0.160
Wooster, Ohio	28	−0.766	±0.078
Warren, Ohio	25	−0.694	±0.071
Roxabel, Ohio	11	−0.743	±0.135
Marietta, Ohio	20	−0.718	±0.108
Jackson, Ohio	26	−0.634	±0.117
Delaware, Ohio	28	−0.588	±0.124
OHIO	**214**	**−0.635**	**±0.027**
Walla Walla, Wash	24	−0.481	±0.104
Kennewick, Wash	24	−0.443	±0.164
Blalock, Wash	18	−0.647	±0.137
Gardena, Wash	19	−0.772	±0.092
Hermiston, Oreg	17	−0.494	±0.183
Milton, Oreg	21	−0.445	±0.175
WASHINGTON AND OREGON	**123**	**−0.478**	**±0.047**
Mather, Wis	23	−0.511	±0.104
For all 16 stations	**360**	**−0.552**	**±0.025**

The correlation coefficient shows a high value, with few exceptions, and strongly indicates a linear relation existing between the two factors considered. The mean correlation coefficient for the 360 observations, embracing all 16 stations, is -0.552 ± 0.025, so that r is here about 28 times as great as E_r.

METHOD OF LEAST SQUARES.

While the above correlations show a close relation between the factors, the practical use of the information can probably be best obtained by a linear equation developed by the method of least squares.

TABLE 7.—*Evening relative humidity, R, and the variation of the minimum temperature of the following morning from the evening dewpoint, Y, at Germantown, Ohio, for those dates in the spring of 1915 when conditions favored radiation.*

Dates.	R	Y	R^2	RY
1915.	%	°F.		
Mar. 12	47	0	2,209	0
13	35	+ 4	1,225	+140
27	53	− 1	2,809	− 53
Apr. 3	53	− 5	2,809	−265
7	44	+ 2	1,936	+ 88
13	52	− 4	2,704	−208
18	30	+ 7	900	+210
24	51	+ 2	2,601	+102
25	63	− 5	3,969	−315
May 1	53	− 4	2,809	−212
9	65	− 8	4,225	−520
10	47	− 2	2,209	− 94
22	67	−12	4,489	−804
24	67	− 7	4,489	−469
June 4	73	−13	5,329	−949
5	70	− 3	4,900	−210
9	53	− 5	2,809	−265
15	64	− 6	4,096	−384
16	64	− 3	4,096	−192
17	54	− 4	2,916	−216
23	39	+ 1	1,521	+ 39
24	38	+10	1,444	+380
25	49	− 1	2,401	− 49
26	47	− 1	2,209	− 47
27	37	+ 3	1,369	+111
Sums (25 = n)	$1,315 = \Sigma R$	$-55 = \Sigma Y$	$72,173 = \Sigma R^2$	$-4,182 = \Sigma RY$

In Table 7 the method of least squares as explained by Prof. C. F. Marvin in the MONTHLY WEATHER REVIEW (loc. cit.), is employed to determine the relation betweeen the relative humidity of the evening and the minimum temperature of the following morning. Therefore, all that seems necessary in explanation of this table is to repeat the formulæ for obtaining the values of a and b, in the general equation

$$y = a + br$$

which here may be written

$$Y = a + bR. \qquad (1)$$

$$b = \frac{n(\Sigma RY) - (\Sigma R)(\Sigma Y)}{n(\Sigma R^2) - (\Sigma R)^2} = -0.39$$

$$a = \frac{\Sigma Y - b(\Sigma R)}{n} = 18.314$$

therefore,

$$y = Y = a + bR = 18.314 - 0.39R.$$

Since R is the relative humidity in the evening, and Y is the departure of the minimum temperature the following morning from the evening dewpoint, it becomes a simple matter to insert the value for R and to compute the value for Y, which becomes the probable departure of the minimum temperature to be expected the following morning from the evening dewpoint. As an example of the application of formula (1) let us substitute for R the relative humidity as observed on the evening of March 12 given in Table 7. The formula now becomes:

$$Y = 18.314 - 0.39 \times 47 = -0.02$$

which indicates that there will be practically no difference between the dewpoint, 23°F., observed on the evening of March 12, and the minimum temperature recorded on the morning of March 13. The temperature actually recorded was 23°F.

Again for the case of May 1, we substitute the value of the evening relative humidity, 50%, in the formula and now get this result:

$$Y = 18.314 - 0.39 \times 53 = -2.4,$$

indicating that the minimum temperature on the morning of May 2 should be 2.4 degrees lower than the dewpoint on the evening of May 1, or 39.6°. The actual temperature recorded was 38° F., or within 1.6 degrees of the estimate.

There is no instance in Table 7 where the value of Y as obtained from the formula varies more than 6 degrees from the actual or recorded value, and a careful investigation would probably reveal other factors not considered that caused the discrepancy, when the variation was so great.

TABLE 8.—*Evening relative humidity, R, and the variation of the minimum temperature of the following morning from the evening dewpoint, Y, at Delaware, Ohio, for those dates in the spring of 1915, when conditions favored radiation.*

Dates.	R	Y	R^2	RY
1915.	%	°F		
Mar. 12	67	− 3	4,489	−201
13	71	− 2	5,041	−142
27	71	− 3	5,041	−213
Apr. 3	57	− 5	3,249	−285
7	65	− 6	4,225	−390
13	57	− 5	3,249	−285
14	51	− 2	2,601	−102
17	42	1	1,764	+ 42
18	60	− 1	3,600	− 60
24	73	− 8	5,329	−584
25	73	− 8	5,329	−584
May 1	74	− 5	5,476	−370
5	60	− 3	3,600	−180
9	73	− 9	5,329	−657
10	65	− 7	4,225	−455
22	78	−12	6,084	−936
24	74	−11	5,476	−814
June 4	90	−11	8,100	−990
5	73	− 2	5,329	−146
9	72	− 9	5,184	−648
15	63	− 8	3,960	−504
16	72	− 9	5,184	−648
17	70	− 9	4,900	−630
23	69	−12	4,761	−828
24	71	− 4	5,041	−284
25	75	− 4	5,625	−300
26	64	− 8	4,096	−512
27	70	−11	4,900	−770
Sums (28 = n)	$1,900 = \Sigma R$	$-176 = \Sigma Y$	$131,196 = \Sigma R^2$	$-12.476 = \Sigma RY$

A similar discussion of the observations at Delaware, Ohio, is presented in Table 8. The value of b is found to be -0.235, and of a to be 9.66. The equation for Delaware then becomes:

$$Y = 9.66 - 0.235R$$

Application of this equation to each date in the spring of 1915 when conditions favored radiation at Delaware, Ohio, showed that in no instance did the computed minimum temperature vary from the actual by more than 5 degrees. As stated in connection with Germantown and Table 7, it is possible that a more careful investigation of these dates when the variation was so great would reveal other contributing factors that have not been taken into account.

There arise the questions (1) whether one may not determine for a and b values that will be constant and applicable to all stations; (2) whether the equation for one station may be applied to another with favorable results. If such is the case, by substituting a constant value for R in each equation, the resultant values for Y should be identical. This was done, 50 being used as the constant R. In Table 9 is given the resulting values for Y whose dissimilarities indicate that these stations present an individual problem in each case. However, it is believed that further study over a longer period of time will establish equations applicable to stations with similar topographic surroundings, and at the same season of the year.

TABLE 9.—*Values of a and b at various stations, and values of Y, determined by using 50 per cent as a constant value for R in equation (1).*

Station.	a	b	Y
Clyde, Ohio	10.149	−0.189	+0.7
Germantown, Ohio	18.314	−0.39	−1.2
Green Hill, Ohio	0.675	−0.108	−4.7
Wooster, Ohio	14.175	−0.289	−0.3
Warren, Ohio	12.786	−0.231	+1.2
Roxabel, Ohio	17.573	−0.261	+2.5
Marietta, Ohio	12.135	−0.332	−4.5
Jackson, Ohio	5.37	−0.196	−4.4
Delaware, Ohio	9.66	−0.235	−2.1
Walla Walla, Wash	12.365	−0.221	+1.3
Kennewick, Wash	9.555	−0.246	−2.7
Gardena, Wash	1.862	−0.071	−1.7
Blalock, Wash	22.615	−0.484	−1.6
Hermiston, Oreg	20.763	−0.573	−7.9
Milton, Oreg	11.816	−0.226	+0.5
Mather, Wis	9.899	−0.226	−1.4

The values of a and b in this table may be of benefit to any who are interested in carrying this subject farther, and who are interested in the hygrometric method of temperature estimates.

Comparison of the hygrometric equation, the night temperature range, and these two in combination.—Recognizing that this hygrometric equation, as developed by the method of least squares, may have its limitations and that too much weight should not be given its use—at least during these experimental stages—a comparison of the results obtained by its use, with those secured by the use of the night temperature range, and by an average between the two methods was made. The comparison appears in Table 10, which checks the computed against the actually recorded minima.

It will be noted of the 25 cases, that in 20 of them the computed minimum was within 3 degrees of the minimum recorded, using the hygrometric method; 13 times as estimated by the night-temperature-range method; and 21 times by the average of the two. Twice the minimum estimated by the hygrometric equation was 6 degrees too low, and once 4 degrees too high. Three times by the night-range method it was 6 degrees too low, and once 7 degrees too high. Twice by the average of the two it was 6 degrees too low and once 5 degrees too high.

It is interesting to note that there is a marked agreement between the minimum estimated by the hygrometric equation and the actual on those dates when frost temperatures occurred, and that the greater departures occurred when the temperature to be expected was not of so much economic importance. This was not true in the temperature-range method.[3]

[3] The agreement here remarked is probably an accidental one.—*C. F. Marvin.*

TABLE 10.—*Minimum temperature estimates for Germantown, Ohio, when conditions favored radiation, using hygrometric method, night-temperature range, and a combination of both, with corresponding errors.*

Dates.	Minimum recorded.	Computed by aid of equation $Y=a+bR$	Error by this method.	Computed by the night-temperature-range method.	Error by this method.	Estimated from mean of first two methods.	Error by this method.
1915.	°F.	°F.	°F.	°F.	°F.	°F.	°F.
Mar. 13	23	23	0	18	−5	20	−3
14	26	27	+1	24	−2	26	0
28	24	23	−1	18	−6	20	−4
Apr. 4	20	23	+3	16	−4	20	0
8	40	39	−1	39	−1	39	−1
14	31	33	+2	28	−3	30	−1
19	44	45	+1	48	+4	46	+2
25	56	52	−4	59	+3	56	0
26	57	56	−1	60	+3	58	+1
May 2	38	40	+2	34	−4	37	−1
10	34	35	+1	39	+5	37	+3
11	40	42	+2	41	+1	42	+2
23	45	49	+4	52	+7	50	+5
25	49	48	−1	50	+1	49	0
June 5	50	53	+3	46	−4	50	0
6	61	55	−6	55	−6	55	−6
10	43	46	+3	43	0	44	+1
16	50	49	−1	51	+1	50	0
17	55	51	−4	54	−1	52	−3
18	57	58	+1	57	0	58	+1
24	43	45	+2	47	+4	46	+3
25	59	53	−6	53	−6	53	−6
26	55	55	0	57	+2	56	+1
27	57	58	+1	58	+1	58	+1
28	54	55	+1	59	+5	57	+3

Table 11 presents, for Delaware, Ohio, a similar comparison of the preceding methods, but adds that of the median-temperature hour. In this table the minimum estimated from the mean of the three methods, hygrometric, night-temperature range, and median-hour, is given in the last two columns.

TABLE 11.—*Minimum temperature estimates for Delaware, Ohio, on dates in spring of 1915 when conditions favored radiation, using hygrometric, night-temperature-range, median-temperature-hour methods, and the mean of these three.*

Dates.	Minimum recorded.	Computed by aid of equation $Y=a+bR$	Error by this method.	Computed by the night-temperature-range method.	Error by this method.	Computed by the median-temperature-hour method.	Error by this method.	Estimated from mean of first three methods.	Error by this method.
1915.	°F.	°F.	°F.	°F.	°F.	°F.	°F.	°F.	°F.
Mar. 13	17	14	−3	13	−4	15	−2	14	−3
14	23	18	−5	20	−3	22	−1	20	−3
28	22	18	−4	14	−8	20	−2	17	−5
Apr. 4	16	17	+1	12	−4	20	+4	16	0
8	28	28	0	33	+5	29	+1	30	+2
14	25	26	+1	27	+2	31	+6	28	+3
15	25	25	0	31	+6	31	+6	29	+4
18	28	27	−1	34	+6	30	+2	30	+2
19	40	37	−3	44	+4	37	−3	39	−1
25	54	55	+1	58	+4	58	+4	57	+3
26	55	56	+1	62	+7	54	−1	57	+2
May 2	36	33	−3	38	+2	35	−1	35	−1
6	38	37	−1	33	−5	40*	+2	37	−1
10	31	33	+2	27	−4	......	......	30†	−1
11	35	36	+1	40	+5	34	−1	37	+2
23	41	44	+3	45	+4	43*	+2	44	+3
25	44	47	+3	45	+1	47	+3	46	+2
June 5	52	52	0	43	−9	56*	+4	50	−2
6	49	44	+5	54	+5	49	0	49	0
10	39	41	+2	41	+2	43	+4	42	+3
16	47	50	+3	48	+1	......	......	49†	+2
17	52	54	+2	51	−1	55	+3	53	+1
18	56	58	+2	56	0	60	+4	58	+2
24	40	45	+5	41	+1	44	+4	43	+3
25	52	49	−3	51	−1	51	−1	50	−2
26	58	54	−4	56	−2	56	−2	55	−3
27	49	52	+3	52	+3	52	+3	52	+3
28	53	57	+4	55	+2	53	0	55	+2

*Allowance made for clouded sky on preceding afternoon by projecting thermograph trace upward.
† *Average of two estimates only.*

It will be noted that at Delaware the estimated minimum by the hygrometric method was within 3 degrees of the actual 22 times out of the 28 cases, and was 5 degrees too low twice and 5 degrees too high once. Using the temperature-range method, the estimate was within 3 degrees of the actual 13 times out of the 28 cases, was once 9 degrees too low, and once 7 degrees too high. In 26 cases using the median-temperature method, 18 times the estimated was within 3 degrees of the actual, twice being 6 degrees too high, and once 3 degrees too low. By using the mean of these three computations we find that in the 28 cases considered the computed was within 3 degrees of the actual 26 times, and that it was once 5 degrees too low and once 4 degrees too high.

CONCLUSIONS.

It seems probable that formulæ may be prepared by two years' thermograph records and whirled psychrometer observations that will permit the making of very accurate minimum temperature forecasts under radiation conditions at any specified point, and further that longer carefully made records will permit of establishing rules that will be widely applicable to places under similar topographic and atmospheric surroundings. The method of hygrometric equation has an advantage in the fact that the dewpoint and relative humidity observations may apparently be made at any convenient hour in the late afternoon or early evening. After the observations have been reduced, and the values of a and b determined, it is then very easy to determine the variation of the minimum temperature from the evening dewpoint by the formula

$$Y=a+bR$$

in which R is the relative humidity at the evening observation and Y the probable variation of the minimum temperature from the evening dewpoint.

HISTORICAL NOTE.

By CHARLES F. MARVIN, Chief of Bureau.

The subject matter of this paper, as regards the prediction of minimum temperatures by means of the so-called median temperature and also the method based on the evening dewpoint and relative humidity, was fully presented by Prof. Smith in a paper submitted August 14, 1915. Publication was deferred at that time because of scantiness of certain data bearing on the question of the change in dewpoint throughout the night, and from other considerations. This lack of data has since been removed and the conclusions of the original paper are herein now seen to be fully confirmed.

Attention is called to the circumstance that as early as 1910 Charles A. Donnel, in analyzing certain special observations made at Boise under the direction of Edward L. Wells, for the purpose of predicting minimum temperatures, noticed that when the evening relative humidity was about 55 per cent, the minimum temperature was about the same as the evening dewpoint. Subsequently, Mr. Wells in a personal report to Mr. Beals, at Portland, Oreg., dated August 9, 1912, stated the relation Donnel had pointed out in the following equation:

M = expected minimum temperature, at orchard level.
D = dewpoint at 8 p. m., at orchard level.
R = relative humidity, at 8 p. m., at orchard level.

Then,

$$M=D-\frac{R-45}{5}$$

This matter was briefly alluded to by Mr. Beals in his bulletin "Forecasting Frost in the North Pacific States," page 17.[4]

During 1917 Floyd D. Young, detailed from the Portland, Oreg., station to manage the spring frost warning service at Medford, Oreg., reported at some length on his utilization of the Donnel relation in that work. Mr. W. G. Reed, also engaged in frost protection investigations at Medford, Oreg., and familiar in a general way with Prof. Smith's investigations, has also studied the utility and application of this relation between the evening hygrometric state and the minimum temperature of the following morning.

It is interesting to observe the close similarity between the entirely independent investigations of Donnel and Wells on the one hand and Prof. Smith on the other. The mathematical identity of the two equations employed is easily shown.

Let M = morning minimum temperature.
Let D = evening dewpoint and R = relative humidity.

DONNEL.	SMITH.
$M=D-\frac{R-n_1}{n_2}$	Y=difference between minimum and dewpoint=$M-D$.
in which n_1, n_2 are two numbers deduced from study of data.	R=evening rel. humidity. a and b are two constants depending on the data.
$M-D=\frac{n_1}{n_2}-\frac{1}{n_2}R$	$Y=a-bR$.
Let $M-D=Y,\ \frac{n_1}{n_2}=a,\ \frac{1}{n_2}=b.$	The computations show that the values of b are negative.
$\therefore\ Y=a-bR$	

551.524 (47+57)

THE LOWEST AIR TEMPERATURE AT A METEOROLOGICAL STATION.

While works on meteorology generally agree in stating that the lowest temperature ever observed at a meteorological station (not including upper-air observations) was recorded at Verkhoyansk, Siberia, the value of the reading and the date of occurrence have been variously stated. The following letter on this subject is in reply to one addressed to the late Prince Boris Galitzin (Russian, Golitsyn), director of the Nicholas Central Physical Observatory, Petrograd. The reference in the last paragraph is to Prof. A. Voeikov's "Meteorologia," St. Petersburg, 1910, p. 73, where the absolute minimum at Verkhoyansk is given as −72°C.—*C. F. Talman, Librarian.*

OBSERVATOIRE PHYSIQUE CENTRAL NICOLAS,
Petrograd, Dec. 2, 1915.

Prof. C. F. MARVIN,
Chief Weather Bureau,
Washington, D. C.

DEAR SIR: In answer to your letter [of] October 4, 1915, about the lowest temperature of air in Verkhoyansk, I can refer you to what follows:

The lowest temperature was noticed by the observer at Verkhoyansk the 5th and 7th of February, 1892. The observations were made by means of the alcohol thermometer Müller No. 81*: the direct reading on this thermometer was −68° C. [=−90.4° F.]. This thermometer, No. 81*, was verified in the Central Physical Observatory, but only

[4] *Beals, E. A.* Forecasting frost in the North Pacific States. Washington, 1912. [12 figs.], 49 p. 8°. (Weather Bureau Bull. 41.) (W. B. No. 473.)